Chapter 1. STATISTICALLY OPTIMAL ADAPTIVE PROCESSING OF 1 AND 3-COMPONENT SMALL APERTURE ARRAY DATA

1. MATHEMATICAL MODELS OF 3-COMPONENT SMALL APERTURE SEISMIC ARRAY RECORDS

Let us suppose hereinafter that seismic waves are generated by a remote seismic source and each of the seismic phases (*P*, *S*, *L*, *R*, etc.) is a plain wave. It is supposed as well that the body wave phases arrive from the homogeneous lower half-space on a surface sequence of laterally homogeneous layers in accordance with unit direction vectors $\boldsymbol{a}_w=(a_{wx},a_{wy},a_{wz})^T$, where *w* represents the wave-type index. We designate as v_w the velocity of *w*-type wave in the half-space directly beneath the layers. Let us assume the origin of coordinates to be placed on the surface of the half-space, with the *Z*-axis directed down, *Y*-axis to the north and *X*-axis to the east; the wave azimuth α is to be counted clockwise from the positive direction of the *Y* axis, and the wave of incidence β_w from the positive direction of the *Z*-axis. Then the vector $\boldsymbol{a}_w$ can be written as $\boldsymbol{a}_w=(\sin\alpha\ \sin\beta_w,\ \cos\alpha\ \sin\beta_w,\ \cos\beta_w)^T$ (^{T} is the sign of transposition). For surface waves $\beta_w=\pi/2$ and $\cos\beta_w=0$.

The displacement $\boldsymbol{w}(t,r)=(w_x(t,r),w_y(t,r),w_z(t,r))^T$ for a particular seismic phase at an arbitrary point $\boldsymbol{r}=(r_x,r_y,r_z)^T$ of the homogeneous half-space can be expressed as follows:

$$\boldsymbol{w}(t,\boldsymbol{r}) = s_w(t - (\boldsymbol{r}^T\boldsymbol{a}_w)/V_w)\boldsymbol{b}_w \tag{1}$$

where $s_w(t)$ is the waveform of a seismic phase at the origin of coordinates, $\boldsymbol{b}=(b_{wx},b_{wy},b_{wz})^T$ is a unit vector of seismic motion which is determined by a the angle of incidence β, azimuth α and the model of transformations of the plain wave at the day surface boundary. For the simplest medium model without consideration of the day surface affecting on the wave field the vector $\boldsymbol{b}$ is expressed by the following simple geometric equations [C.Emersoy et al., 1985]

for P-waves
$$\boldsymbol{b}_P = \begin{bmatrix} \sin\alpha\sin\beta \\ \cos\alpha\sin\beta \\ \cos\beta \end{bmatrix}; \tag{2a}$$

for SH-waves and for Love waves
$$\boldsymbol{b}_L = \begin{bmatrix} \cos\alpha \\ \sin\alpha \\ 0 \end{bmatrix}; \tag{2b}$$

for SV-waves
$$\boldsymbol{b}_V = \begin{bmatrix} \sin\alpha\cos\beta \\ \cos\alpha\cos\beta \\ \sin\beta \end{bmatrix}; \tag{2c}$$

for Rayleigh waves $$\boldsymbol{b}_R = \begin{bmatrix} i\,sin\,\alpha\,sin\,\beta \\ i\,cos\,\alpha\,sin\,\beta \\ cos\,y \end{bmatrix}; \qquad (2d)$$

where $\psi = arctg(e)$, e characterizes the elliptic feature of the Rayleigh wave, e.g. the ratio of the small axis of the polarization ellipse to the large one; $i = \sqrt{-1}$ represents the phase shift of $\pi/2$ between the vertical and horizontal components of Rayleigh wave displacements.

In the frequency domain eq.(1) has the form

$$\boldsymbol{w}(f,\boldsymbol{r}) = s_W\,(f)\,exp[-i2\pi\,f(\boldsymbol{r}^T\,\boldsymbol{a}_W\,)/V_W\,]\;\boldsymbol{b}_W$$

(3)

where $s_W(f)$ is the complex spectrum of the waveform. We further introduce the 3-dimensional slowness vector $\boldsymbol{p}_W=(p_X,\,p_y,\,p_Z)^T = \boldsymbol{a}/V_W$. Then the wave field eq.(2) can be expressed as a function of the vector $\boldsymbol{p}_W$ as follows:

$$\boldsymbol{w}(f,\boldsymbol{r}) = s_W\,(f)\,exp[-i2\pi\,f(\boldsymbol{r}^T\,\boldsymbol{p}_W\,)]\;\boldsymbol{b}(\,\boldsymbol{p}_W, V_w)$$

(4)

Dependence of the vector $\boldsymbol{b}$ from the vector $\boldsymbol{p}_w$ in eq.(4) is based on the following simple geometric relations:

$$sin\alpha = p_x/p_h; \quad cos\alpha = p_y/p_h; \quad sin\beta_W = p_h V_W; \quad p_h = (p^{-2}{}_X + p^{-2}{}_y\,)^{-1/2}.$$

(5)

Equations (2) and (5) imply that for the simplest media model the vector $\boldsymbol{b}$ depends <u>only</u> on values of the apparent velocities p_x, p_y, p_h and phase velocities V_p, V_s via the following equations:

for P-waves $$\boldsymbol{b}_P = \begin{bmatrix} p_x \\ p_y \\ \sqrt{V_p^{-2} - p_h^2} \end{bmatrix} V_P$$

(6a)

for SH-waves and Love waves $$\boldsymbol{b}_L = \begin{bmatrix} p_y \\ p_x \\ 0 \end{bmatrix} .\, p_h^{-1}$$

(6b)

for SV - waves $$\boldsymbol{b}_V = \begin{bmatrix} p_x \\ p_y \\ p_h^2 \big/ \sqrt{V_s^{-2} - p_h^2} \end{bmatrix} \sqrt{p_h^{-2} - V_s^2}$$

(6c)

for Rayleigh waves $$\boldsymbol{b}_R = \begin{bmatrix} p_x \\ p_y \\ -i \quad ctg\,\psi \quad p_h \end{bmatrix} .ip_h^{-1}\,sin\,\psi \qquad (6d)$$

Note that for Rayleigh and Love waves p_x and p_y depend on frequency *f*.

2. NOISE SUPPRESSION AND SEISMIC WAVEFORM EXTRACTION USING DATA FROM 3-COMPONENT SMALL APERTURE ARRAYS

2.1. Introduction

The 3-component small aperture arrays are the subject of the primary interest as the Alpha-stations of the International Monitoring Network being developed for verification of the Test Ban Treaty. The usage of 3-component array data allows to significantly enhance the quality of extraction of seismic phase waveforms from a background noise especially if to employ the adaptive optimal group filtering (AOGF) method. The possibilities are exists to extract waveforms of different event phases that are characterized by different polarization features, including those phases that can not be handled using only 1-component array data, such as regional SH and teleseismic Love.

The seismic noise (especially in the sea-shore regions) often is the transient one and is constituted by the waves generated by a surf or industrial sources. In these cases it exhibits explicit coherent and polarization features. As shown in publications of the report authors, the noise coherency can be successfully utilized by the AOGF method for the case of 1-component array data processing. This method can be expanded for 3-component (3C) array data processing. In the last case the (*3m x 3m*) matrix power spectral density (MPSD) of 3C noise records have to be estimated and the polarization characteristics of noise are automatically captured in this MPSD. To adjust correctly the AOGF for an extraction of waveform of given seismic phase waveform one have to account the polarization features of this phase at a site of 3C station. So different phases should be treated by the different AOG filters. Parallel processing of the same 3-component array recordings by these filters produces three output traces that reflects wavetrain oscillations in the longitudinal, transverse and vertical direction generated by the P, SH and SV body waves; Lg, Rayleigh and Love surface waves. Such analysis can be helpful for investigation of complex wave-fields in regions with strong laterally heterogeneous media structure. It can be implemented for enhancing of event source location and identification quality in regional monitoring with the help of a 3-component arrays.

Some complexity of computing while the 3C array data processing is justified by the advantages of the combined procedure described above that accounts for differences in both: the relative delays and the polarization characteristics of array signals and noise. The application of this procedure can provide for our assessment the same quality of signal extraction from data of 3C small aperture array as from data of 1C small aperture array with 2 - 3 times larger amount of sensors.