Chapter 5. NUMERICAL METHOD FOR THE SOLUTION OF THE INVERSE PROBLEM FOR DETERMINATION OF SOURCE PARAMETERS

1. INTRODUCTION

The most reliable algorithm for seismic source type identification according to the earthquake-explosion criterion could be one based directly on estimation of the physical processes in the event source. In this respect the most comprehensive decision is to estimate from seismic field recordings an evolution in time of all the six independent components of the seismic moment tensor of the equivalent event source. It allows to carry out the complete analysis of dynamic, spatial and spectral features of the seismic source, evaluate its shear and cleavage stress components, and thus reliably determine the seismic source type. Such approach to identification of sources requires solving the inverse problem of the elasticity dynamic theory using records of high-frequency body wave field of a seismic event.

The inverse problem approach permits to measure from seismic observations the essentially new dynamic characteristics of explosion and earthquake sources – the time dependent shear, cleavage and hydrostatic stresses and also the time variations of orientations of the maximal tangency surface elements. This allows fulfilling the identification of explosions and earthquakes on a new more physically authentic level.

2. STATEMENT OF THE PROBLEM OF INVERSE SEISMIC MOMENT TENSOR

A system of the Lame equations is considered which describes the motion of the homogeneous ideally elastic medium:

$$\frac{\partial \sigma_{ij}}{\partial x_j} - \rho \frac{\partial^2 u_i}{\partial t^2} = M_{ij}(t) \frac{\partial}{\partial x_j} \delta(x-y). \qquad (1)$$

Here $i, j = 1,2,3,\ x, y \in R^3;\ t \in R^1;\ \rho$ is the medium density, σ_{ij} is the stress tensor related to the displacement vector $u(x,t) = (u_1, u_2, u_3)$ in the form

$$\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda \delta_{ij} \frac{\partial u_k}{\partial x_k},$$

where λ, μ are the Lame constants and the repetition of indices means summation.

The symmetric tensor $M_{ij}(t)$ of the rank 2 is called the seismic moment tensor and has dimensionality in energy units $(g \cdot cm^2 \cdot s^{-1})$. In terms of equivalent forces, it describes shift discontinuity, shift destruction of the medium, transformational change of the volume, crack of separation, etc. [1]. Let the parameter t_0 characterize the start moment of the process in the source $(M_{ij}(t) \equiv 0, t < t_0)$ Let us also suppose that λ, μ and ρ are the known constants.

The inverse problem consists of determination of the parameters t_0, y and the tensor $M_{ij}(t)$ from the data having the form

$$v_k(t) = u(x_k, t) + \varepsilon_k(t), \qquad k \geq 4. \tag{2}$$

Here $x_k \in R^3$, ε_k is the noise with the normal probability distribution, zero mean value and some known covariance matrix $G_\varepsilon(x_k, x_{k'})$.

3. DESCRIPTION OF ALGORITHM FOR SOLUTION OF THE INVERSE PROBLEM

Let us assume that there exists a formal procedure A with the help of which we may determine, using the data of form (2), either the arrival times of P-waves and S-waves separately, or only the difference of the arrival times of P-waves and S-waves.

$$A\{v_k(t)\} = t_p^k, t_s^k, t_s^k - t_p^k. \tag{3}$$

It is reasonable to design the solution of inverse problem (1) - (3) as having *two steps*: at the *first step* we determine from data (3) the parameters t_0, y; at the *second step* we determine from data (2) the dynamic characteristics of the source, that is the tensor $M_{ij}(t)$, providing that its coordinate y and the start time t_0 of the process are known.

The both steps of the problems are solved by minimizing the functional of the form:

$$J(\eta) = (w - F(\eta))^T V(w - F(\eta\}\} + \Omega(\eta\} + Sh(\eta). \tag{4}$$

Here $w = (w_{1,} \ldots, w_N)$ is the vector of measurements determined from data (2) and (3), $\eta = (\eta_1, \ldots, \eta_n)$ is the vector of the desired parameters; $F(\eta)$ is the vector-function of the solution derived from equation (1) having the same dimension as for the data $w; (\cdot)^T$ denotes the line vector; V is a positively determined symmetric matrix which can coincide with G_ε^{-1}, and $Sh(\eta)$ is some external penalty functional. The functional Ω in (4), in accordance with A.N. Tikhonov results [2], is the stabilizing functional:

$$\Omega(\eta\} = (\eta - \eta_0)^T G(\eta - \eta_0\}. \tag{5}$$

Here G is some positively defined symmetric $n \times n$- matrix of the regularity coefficients, η_0 is the vector of a priori values of the stabilizing functional. It should be noted that if we know the covariance matrix $G_\eta = \langle \eta\eta' \rangle$ of the parameters, then we can take $G = G_\eta^{-1}$.

4. THE METHOD OF MINIMIZATION

The minimum of functional (3) is determined by the technique which is the combination of the Marquardt method and the regularized quasi-Newtonian method using only the first derivatives of the functional. The step of controlled iterative minimization procedure is given by following formulas:

$$\eta^{k+1} = \eta^k + \alpha_k p^k \tag{6}$$

$$p^k = -A^{-1}(\eta^k, \zeta^k) Y(\eta^k, \zeta^k). \tag{7}$$

Here α_k is the step of the parameter regulation, $1 \le k \le k_{\max}$,

$$Y = \left(\frac{\partial F}{\partial \eta} \right)^T V(F - w) + G(\eta - \eta_0) + \frac{\partial \ Sh}{\partial \ \eta}, \tag{8}$$

$$A = \left(\frac{\partial F}{\partial \eta} \right)^T V \left(\frac{\partial F}{\partial \eta} \right) + \Lambda + \frac{\partial^2 Sh}{\partial^2 \eta}. \tag{9}$$

The vector ζ denotes the combination of all the free parameters of functionals (4)--(5) and iterative processes (6)--(9) being adjusted at each step with respect to k:

$$\zeta^{k+1} = \zeta^k + \Delta\zeta \ (\eta^1, \ldots, \eta^k).$$

Thus, while solving inverse problem (1)--(3), the adaptive approach is used when the penalty functional parameters and the Tikhonov's stabilizer are formed at each iterative step. The methods are combined by introducing the independent matrices Λ and G. In fact, if $\Lambda = G$, we obtain the regularization of the quasi-Newtonian method. When $G=0$, algorithm (6)--(9) is the generalization of the Marquardt method (here Λ is an arbitrary matrix, in contrast to the conventional technique where $\Lambda = \lambda I$).

The parameters α_k in (6) at each step of the iterative process are selected as the maximal parameters from the set $\{\alpha_k | 0 < \alpha_k < \alpha_0\}$, so that the inequality

$$J(\eta^k + \alpha_k p^k) - J(\eta^k) \le \varepsilon \alpha_k (Y, p^k),$$

where $0 < \varepsilon \le \frac{1}{2}$, is fulfilled.

When $\alpha_0 = 1$, the method with the step-wise regularization coincides with the conventional method of minimization. In the case of $\alpha_0 < 1$, the search for the global minimum in the direction of the anti-gradient is carried out. If this global minimum is not found, the standard technique of subdividing α is used.

Iterative process (6)--(9) is finished if $k = k_{\max}$, or if the absolute values of differences between the values of several parameters become smaller than some preset values $\delta_{i,}$ $i = 1, \ldots, n$.